

3.5 | Heat Capacities of an Ideal Gas

Learning Objectives

By the end of this section, you will be able to:

- Define heat capacity of an ideal gas for a specific process
- Calculate the specific heat of an ideal gas for either an isobaric or isochoric process
- Explain the difference between the heat capacities of an ideal gas and a real gas
- Estimate the change in specific heat of a gas over temperature ranges

We learned about specific heat and molar heat capacity in **Temperature and Heat**; however, we have not considered a process in which heat is added. We do that in this section. First, we examine a process where the system has a constant volume, then contrast it with a system at constant pressure and show how their specific heats are related.

Let's start with looking at **Figure 3.12**, which shows two vessels A and B, each containing 1 mol of the same type of ideal gas at a temperature T and a volume V . The only difference between the two vessels is that the piston at the top of A is fixed, whereas the one at the top of B is free to move against a constant external pressure p . We now consider what happens when the temperature of the gas in each vessel is slowly increased to $T + dT$ with the addition of heat.

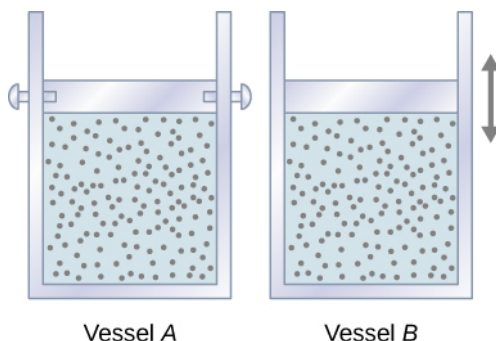


Figure 3.12 Two vessels are identical except that the piston at the top of A is fixed, whereas that atop B is free to move against a constant external pressure p .

Since the piston of vessel A is fixed, the volume of the enclosed gas does not change. Consequently, the gas does no work, and we have from the first law

$$dE_{\text{int}} = dQ - dW = dQ.$$

We represent the fact that the heat is exchanged at constant volume by writing

$$dQ = C_V dT,$$

where C_V is the **molar heat capacity at constant volume** of the gas. In addition, since $dE_{\text{int}} = dQ$ for this particular process,

$$dE_{\text{int}} = C_V dT. \quad (3.9)$$

We obtained this equation assuming the volume of the gas was fixed. However, internal energy is a state function that depends on only the temperature of an ideal gas. Therefore, $dE_{\text{int}} = C_V dT$ gives the change in internal energy of an ideal gas for any process involving a temperature change dT .

When the gas in vessel B is heated, it expands against the movable piston and does work $dW = pdV$. In this case, the heat is added at constant pressure, and we write

$$dQ = C_p dT,$$

where C_p is the **molar heat capacity at constant pressure** of the gas. Furthermore, since the ideal gas expands against a constant pressure,

$$d(pV) = d(RT)$$

becomes

$$pdV = RdT.$$

Finally, inserting the expressions for dQ and pdV into the first law, we obtain

$$dE_{\text{int}} = dQ - pdV = (C_p - R)dT.$$

We have found dE_{int} for both an isochoric and an isobaric process. Because the internal energy of an ideal gas depends only on the temperature, dE_{int} must be the same for both processes. Thus,

$$C_V dT = (C_p - R)dT,$$

and

$$C_p = C_V + R. \quad (3.10)$$

The derivation of **Equation 3.10** was based only on the ideal gas law. Consequently, this relationship is approximately valid for all dilute gases, whether monatomic like He, diatomic like O_2 , or polyatomic like CO_2 or NH_3 .

In the preceding chapter, we found the molar heat capacity of an ideal gas under constant volume to be

$$C_V = \frac{d}{2}R,$$

where d is the number of degrees of freedom of a molecule in the system. **Table 3.3** shows the molar heat capacities of some dilute ideal gases at room temperature. The heat capacities of real gases are somewhat higher than those predicted by the expressions of C_V and C_p given in **Equation 3.10**. This indicates that vibrational motion in polyatomic molecules is significant, even at room temperature. Nevertheless, the difference in the molar heat capacities, $C_p - C_V$, is very close to R , even for the polyatomic gases.

Molar Heat Capacities of Dilute Ideal Gases at Room Temperature				
Type of Molecule	Gas	C_p (J/mol K)	C_V (J/mol K)	$C_p - C_V$ (J/mol K)
Monatomic	Ideal	$\frac{5}{2}R = 20.79$	$\frac{3}{2}R = 12.47$	$R = 8.31$
Diatomic	Ideal	$\frac{7}{2}R = 29.10$	$\frac{5}{2}R = 20.79$	$R = 8.31$
Polyatomic	Ideal	$4R = 33.26$	$3R = 24.94$	$R = 8.31$

Table 3.3